

(SU(2) Triplet)

Extended Higgs Sector from GUTs and EWSB

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17th Workshop on Weak Interactions and Neutrinos
Lake Geneva, Wisconsin, Oct 6-11, 2003

The plan:

- (i) motivation
- (ii) extended Higgs sector from GUTs and SB
- (iii) EW constraints
- (iv) what is the triplet Higgs good for
- (v) signatures
- (vi) open questions

LEP

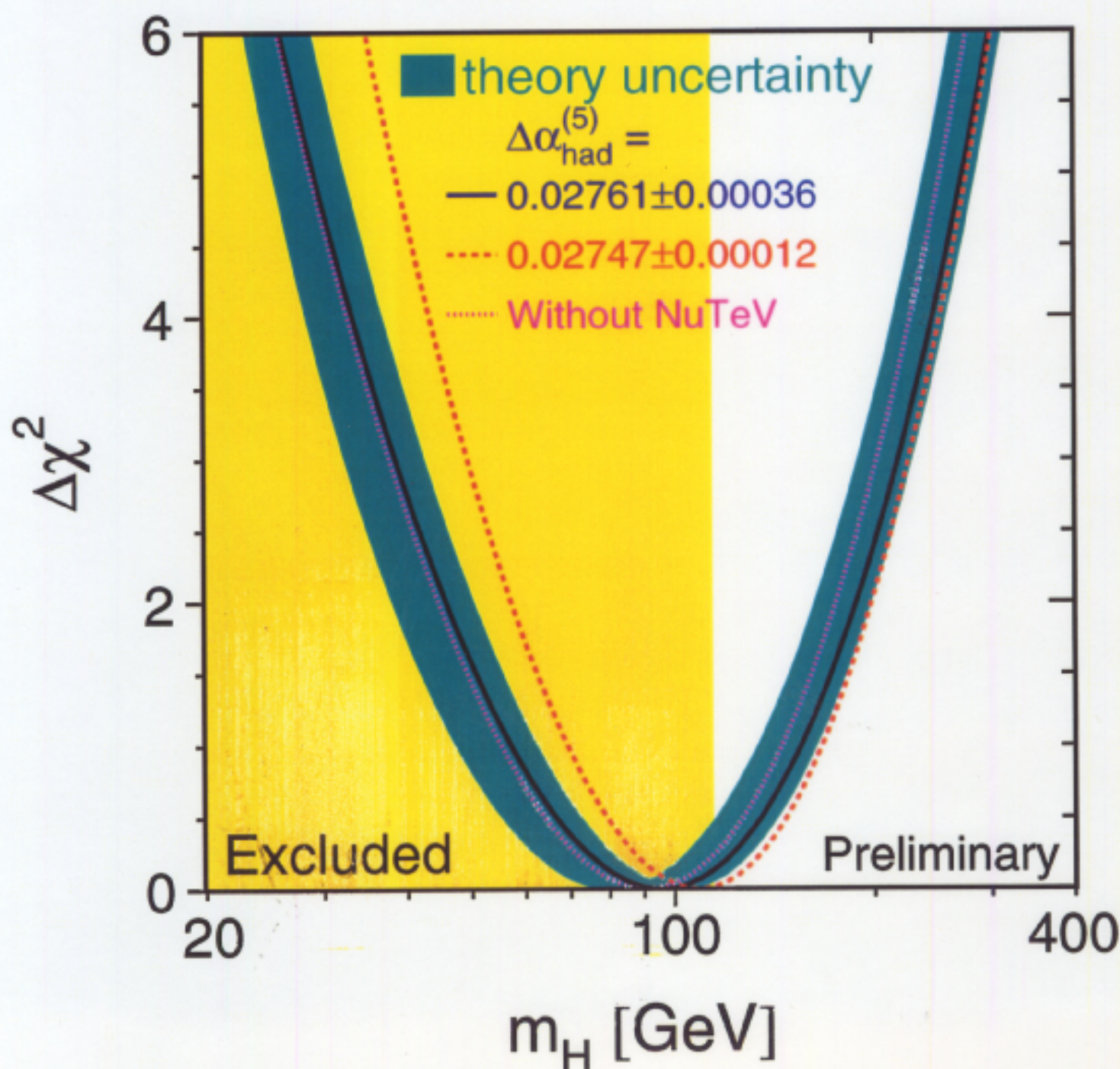
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Some Hints from current data:

Global fit including all observables with

$\Rightarrow m_H \approx 96 \text{ GeV}$ is preferred ($81 \pm {}^{52}_{33} \text{ GeV}$)

\Rightarrow lower than current limit from LEP: $> 114 \text{ GeV}$



Why Extended Higgs Sector?

SM Higgs is predicted to be light, yet we have not found it!

There are several ways to evade the lower bound from LEP data: (Peskin and Wells, 2001)

$$\Delta T > 0, \quad \Delta S < 0$$

- Specific low energy effective models that have been looked at
 - $\Delta T > 0$
 - * 2 Higgs doublets (Chankowski et al)
 - * 4th generation (Dobrescu and Hill; He et al; ...)
 - $\Delta S < 0$
 - * extra singlet Majorana fermions (Gates and Turning)
 - * extra $SU(2) \times SU(2)$ multiplets (Dugan and Randall)
- extended scalar sector:
 - 4D GUT Models: lots of exotic scalars
 - GUTs in higher dimensions

Orbifold boundary conditions can only break non-abelian symmetry: left over $U(1)$'s

gauge symmetry breaking above EW scale \Rightarrow
by orbifold boundary conditions

EWSB \Rightarrow by conventional Higgs mechanism;
dynamical SB

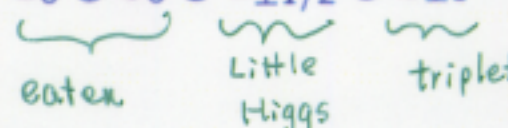
\Rightarrow much simpler Higgs sector compared to conventional 4D GUT models

– Little Higgs Models:

Higgs as a pseudo-Goldstone boson

Littlest Higgs Model: $SU(5)/SO(5)$

$$14 \rightarrow 1_0 \oplus 3_0 \oplus 2_{\pm 1/2} \oplus 3_{\pm 1}$$


eaten Little Higgs triplet

– Randall-Sundrum Model: Radion

Unification can also be achieved without SUSY by adding the following choices of Higgs representations $N_{T,Y}$ to the SM

(J. Gunion, hep-ph/0212150)

$N_{1/2,1}$	$N_{1/2,3}$	$N_{0,2}$	$N_{0,4}$	$N_{1,0}$	$N_{1,2}$	$\alpha_s(M_z)$	M_G (GeV)
1	0	0	2	0	0	0.106	4.0×10^{12}
1	0	4	0	0	1	0.112	7.7×10^{12}
1	0	0	0	0	2	0.120	1.6×10^{13}
2	0	0	0	1	0	0.116	1.7×10^{14}
2	0	2	0	0	2	0.116	4.9×10^{12}
2	1	0	0	0	2	0.112	1.7×10^{12}
3	0	0	0	0	1	0.105	1.2×10^{13}

\Rightarrow lower unification scale compared to $M_{GUT} \sim 2 \times 10^{16} \text{ GeV}$ in typical SUSY GUT scenario

\Rightarrow proton decay NOT a problem, as there are NO X, Y gauge bosons, if not imbedded into a single gauge group (as in some string models)

Nonetheless, no predictivity !!

Extended Higgs Structure from GUT Models

- Left-Right Symmetric Models: $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$

$$\text{Typically } \begin{array}{l} Q_L \sim (2, 0) \\ l_L \sim (2, 0) \end{array}, \quad \begin{array}{l} Q_R \sim (0, 2) \\ l_R \sim (0, 2) \end{array} \quad \left| \quad Q_{EM} = I_{3L} + I_{3R} + \frac{B-L}{2} \right.$$

$$1 \text{ bi-doublet } \Phi = \begin{pmatrix} \phi_1^+ & \phi_2^+ \\ \phi_1^0 & \phi_2^0 \end{pmatrix} \sim (2, 2)$$

$$2 \text{ complex triplets } \Delta_L = (\Delta_L^{++}, \Delta_L^+, \Delta_L^0) \sim (3, 1)$$

$$\Delta_{L,R} = \begin{pmatrix} \Delta_{L,R}^{++}/\sqrt{2} & \Delta_{L,R}^{++} \\ \Delta_{L,R}^0 & -\Delta_{L,R}^+/ \sqrt{2} \end{pmatrix} \quad (Y=2) \quad \Delta_R = (\Delta_R^{++}, \Delta_R^+, \Delta_R^0) \sim (1, 3)$$

– Symmetry breaking:

$$SU(2)_L \times SU(2)_R \times U(1)_{B-L} \xrightarrow{\langle \Delta_R \rangle} SU(2)_L \times U(1)_Y \xrightarrow{\langle \phi_1 \rangle} U(1)_{ew}$$

– Mass Spectrum:

$$\langle \Delta_R^0 \rangle \gg v_{EW}, \quad M_{\tilde{L}}, M_{W_R^{\pm}} \sim v_R^2 \rightarrow \text{heavy}$$

$$M_{\Delta_R} \sim M_{\Delta_L} \sim v_R$$

– Gauge coupling constant unification??

require other matter fields at intermediate scale

Lindner and Weiser, 1996

need extra dimensions Perez-Lorenzana, Ponce, Zepeda, 1999; Perez-Lorenzana and Mohapatra, 1999

- $SO(10)$ Models:

Minimal Higgs Sector ($G_{10} \rightarrow G_{2,2,4} \rightarrow G_{3,2,1}$)

$$10 = (1, 1, 6) + (2, 2, 1)$$

$$16 = (2, 1, 4) + (1, 2, \bar{4})$$

$$45 = \underline{(3, 1, 1)} + (2, 2, 6) + (1, 1, 15) + (1, 3, 1)$$

$$54 = (1, 1, 1) + (2, 2, 6) + (1, 1, 20) + (3, 3, 1)$$

For Majorana masses of ν_R :

$$126 = \underline{(3, 1, 10)} + (1, 3, \bar{10}) + (2, 2, 15) + (1, 1, \bar{6})$$

Lots of exotic stuff!! They must be heavy, otherwise could lead to bad consequences, e.g. proton decay mediated by color triplet Higgsino (dim-5 operator) in SUSY $SO(10)$ – "doublet-triplet splitting problem"

(MS)SM Higgs doublet(s): linear combination(s) of SU(2) doublet components in 10, 16, and/or 126

The bottom line is:

Non-SUSY GUTs: can have light scalar fields in addition to the SM Higgs; nonetheless predict low unification scale

SUSY GUTs: to preserve unification, require all but MSSM Higgs doublets heavy $\sim M_{GUT}$

From now on, concentrate on light Triplet Higgs and the Left-Right Symmetry Group

EW Precision Constraints

Oblique Corrections:

$$S = \frac{4s_w^2 c_w^2}{M_z^2} (\Delta \Pi^{zz}(M_z) - \frac{c_w^2 - s_w^2}{s_w c_w} \Delta \Pi^{\gamma z}(M_z) - \Delta \Pi^{\gamma\gamma}(M_z))$$

$$T = \frac{1}{M_w^2} (\Pi^{ww}(0) - \Pi^{zz}(0) c_w^2)$$

$$U = 4s_w^2 \left(\frac{\Delta \Pi^{ww}(M_w)}{M_w^2} - \frac{c_w}{s_w} \frac{\Delta \Pi^{\gamma z}(M_z)}{M_z^2} - \frac{\Delta \Pi^{\gamma\gamma}(M_z)}{M_z^2} \right)$$

Very Model Dependent. Here are two examples:

- SM with a real SU(2) Triplet Higgs ($Y = 0$)
(Blank and Hollik, 1998; Forshaw et al, 2001, 2003)

The Lagrangian:

$$\mathcal{L} = |Dh|^2 + \frac{1}{2} |D\Delta|^2 - V_0(h, \Delta) \quad , \quad \Delta = \begin{pmatrix} \Delta^+ \\ \Delta^0 \\ \Delta^- \end{pmatrix}$$

The scalar potential is

$$V_0 = \mu_1^2 |h|^2 + \frac{\mu_2^2}{2} |\Delta|^2 + \lambda_1 |h|^4 + \frac{\lambda_2}{2} |h|^2 |\Delta|^2 + \lambda_3 |\Delta|^4 + \lambda_4 h \Delta h^\dagger$$

$\phi^0 \rightarrow Z^0$

$$h = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}} v + \phi_R^0 + i \phi_i^0 \end{pmatrix} \quad , \quad \langle \Delta \rangle = \begin{pmatrix} \eta^+ \\ \frac{1}{\sqrt{2}} v_\Delta \epsilon_\beta + \eta^0 \\ -\eta^- \end{pmatrix}$$

$g^\pm \rightarrow W^\pm$

$$\begin{pmatrix} g^\pm \\ h^\pm \end{pmatrix} = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} \phi^\pm \\ \eta^\pm \end{pmatrix} \quad , \quad \tan \beta = 2 \frac{\langle \Delta \rangle}{\langle h \rangle}$$

$$\begin{pmatrix} H^0 \\ A^0 \end{pmatrix} = \begin{pmatrix} \cos \gamma & \sin \gamma \\ -\sin \gamma & \cos \gamma \end{pmatrix} \begin{pmatrix} h^0 \\ \eta^0 \end{pmatrix}$$

$$M_h \simeq \lambda_1 v^2$$

$$M_{\Delta^0} \simeq M_{\Delta^\pm} \simeq \lambda_+ v^2 / \beta, \quad \beta \rightarrow 0, \quad \text{custodial symm. restored}$$

$$\Delta m = M_{\Delta^0} - M_{\Delta^\pm} \simeq \beta^2 v$$

at tree level:

$$M_W = \frac{g^2}{4}(v^2 + 4v_{1,0}^2), \quad M_Z = \frac{g^2 + g'^2}{4}v^2$$

Thus the model predicts

$$\rho^{\text{tree}} = 1 + \frac{4v_{1,0}^2}{v^2} = \frac{1}{\cos^2 \beta} > 1$$

β = mixing angle between the charged components of the doublet and the triplet

at one-loop:

$$\Delta S^{\text{tri}} = 0, (Y = 0)$$

$$\begin{aligned} \Delta T^{\text{tri}} &= \frac{1}{8\pi^2} \frac{1}{s_w^2 c_w^2} \left[\frac{m_0^2 + m_c^2}{M_z^2} - \frac{2m_0^2 m_c^2}{M_z^2 (m_0^2 - m_c^2)} \ln\left(\frac{m_0^2}{m_c^2}\right) \right] \\ &\sim \frac{1}{6\pi} \frac{1}{s_w^2 c_w^2} \frac{(\Delta m)^2}{M_z^2} \end{aligned}$$

SM one-loop contributions:

$$\begin{aligned} \rho^{\text{Higgs}} &= \frac{3}{16\pi^2} \frac{1}{s_w^2 c_w^2} \left[\frac{m_h^2}{M_z^2 - m_h^2} \ln\left(\frac{m_h^2}{M_z^2}\right) - \frac{m_h^2 c_w^2}{M_z^2 c_w^2 - m_h^2} \ln\left(\frac{m_h^2}{M_w^2}\right) \right] \\ &\sim -\ln\left(\frac{m_h^2}{M_w^2}\right) \end{aligned}$$

The effects of the triplet contributions > 1 (tree level)
 \Rightarrow making heavy SM Higgs possible!

$$\boxed{\beta \lesssim 4^\circ}$$

For all EW precision observables: (Blank & Hollik)

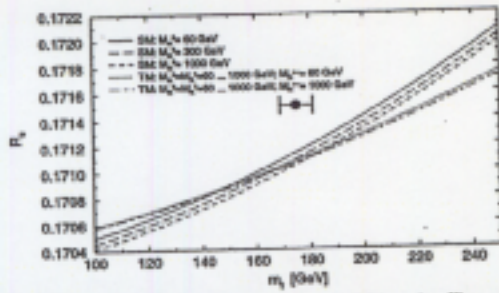


Figure 5.8: Top mass dependence of R_b in the SM and the TM for various Higgs masses. The error bar of R_b covers the full vertical axis.

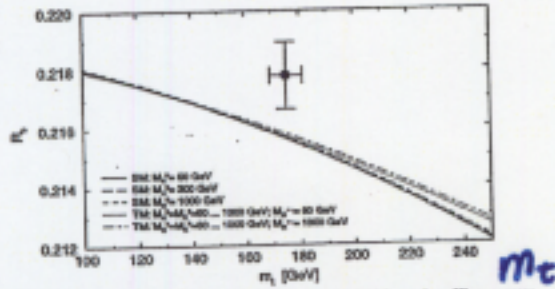


Figure 5.9: Top mass dependence of R_b in the SM and the TM for various Higgs masses.

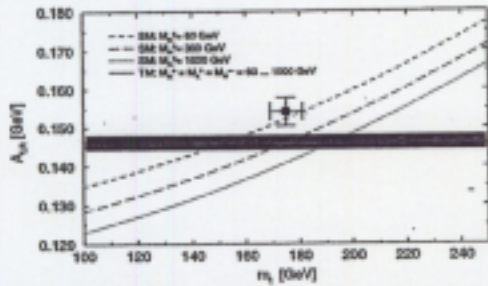


Figure 5.10: Left/Right asymmetry in the SM and the TM. The shaded area corresponds to a variation of $\alpha_s^2 = 0.23165 \pm 0.00034$.

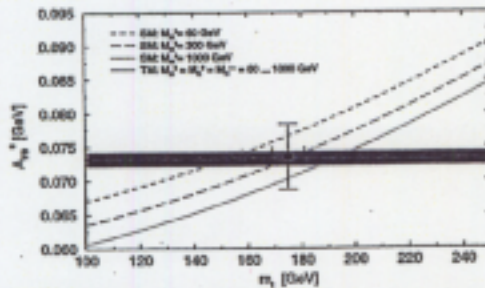


Figure 5.11: Forward/backward asymmetry for charm quarks in the SM and the TM. The shaded area corresponds to a variation of $\alpha_s^2 = 0.23165 \pm 0.00034$.

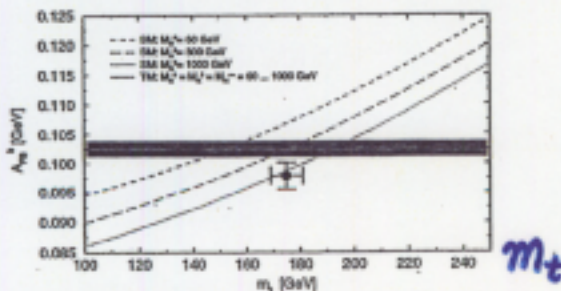


Figure 5.12: Forward/backward asymmetry for bottom quarks in the SM and the TM. The shaded area corresponds to a variation of $\alpha_s^2 = 0.23165 \pm 0.00034$.

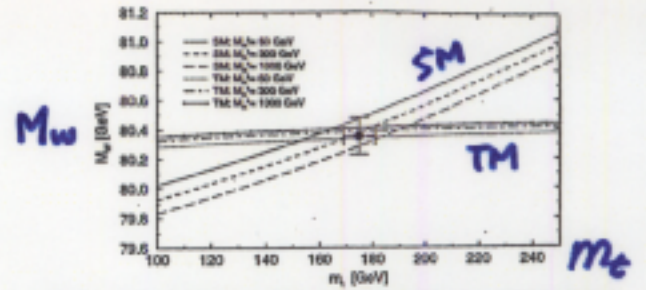


Figure 5.1: Top mass dependence of M_W in the SM and the TM for various doublet Higgs masses M_H . The input values for the TM Higgs masses M_{H_1} and M_{H_2} are 300 GeV.

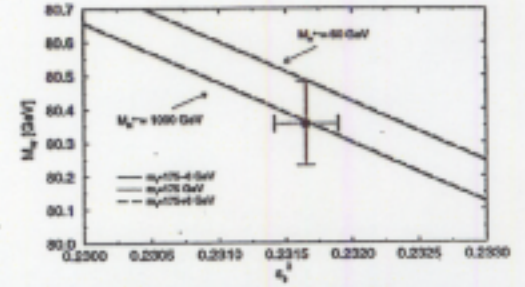


Figure 5.2: Dependence of M_W on the input parameter α_s^2 for various values of m_t and M_H in the TM. The masses for the neutral Higgs bosons are fixed at 300 GeV.

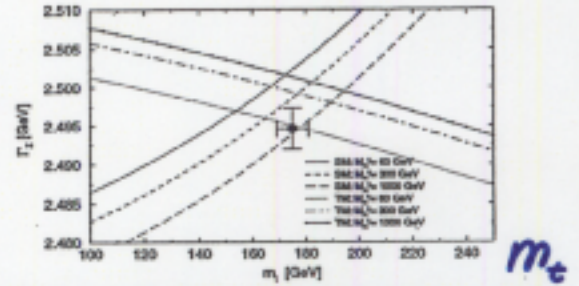


Figure 5.3: Top mass dependence of the total Z width in the SM and the TM for various doublet Higgs masses M_H . The input values for the TM Higgs masses M_{H_1} and M_{H_2} are 300 GeV.

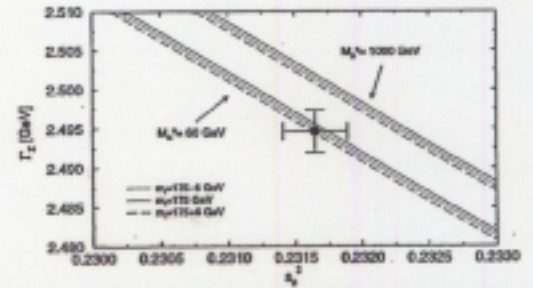


Figure 5.4: Dependence of the total Z width on the input parameter α_s^2 for various values of m_t and M_H . The masses of the triplet Higgs bosons are fixed at 300 GeV.

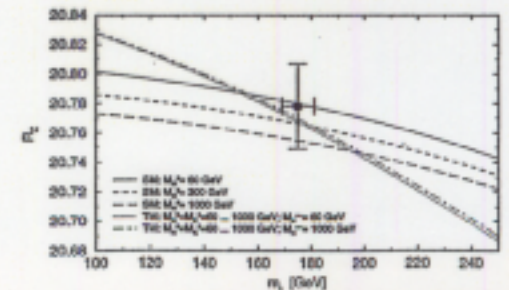


Figure 5.5: Top mass dependence of R_Z in the SM and the TM for various Higgs masses.

Predictions for all observables coincide with SM predictions, which fully agree with experiment except for R_b and A_{FB}^b :

Both models show similar deviation from data

Require quartic coupling constants for both SM Higgs and the triplet perturbative up to $\Lambda \sim 1\text{TeV}$:

$$\begin{aligned} m_h &< 520\text{GeV}, & (\lambda_1|h|^4) \\ m_\Delta &< 550\text{GeV}, & (\tfrac{1}{2}\lambda_2|h|^2|\Delta|^2) \end{aligned}$$

To pinpoint the mass of the SM Higgs via the indirect method: need to determine S independently of T better than ± 0.1 (Rosner, 2002)

- Left-Right Symmetric Model
(Jegerlehner et al, 2000)

$$SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

model contains **a** complex $SU(2)$ Triplet Higgses
($Y = 2$) and heavy gauge bosons, **1** bi-doublet

$$\langle \Delta_{L,R} \rangle = \begin{pmatrix} 0 & 0 \\ v_{L,R}/\sqrt{2} & 0 \end{pmatrix}$$

$$\langle \vec{q} \rangle = \begin{pmatrix} K_1/\sqrt{2} & 0 \\ 0 & K_2/\sqrt{2} \end{pmatrix}, \quad \text{set } K_2 = 0$$

$$v_R \gg K_1 \sim v_{EW} \gg v_L \sim \frac{v_{EW}^2}{v_R}$$

at tree level: $\langle \Delta_L^0 \rangle = v_{1,-1}$

$$M_W = \frac{g^2}{4}(v^2 + 2v_{1,-1}^2), \quad M_Z = \frac{g^2 + g'^2}{4}(v^2 + 4v_{1,-1}^2)$$

Thus the model predicts

$$\rho^{\text{tree}} = \frac{v^2 + 2v_{1,-1}^2}{v^2 + 4v_{1,-1}^2} < 1$$

at one loop:

Recall that in SM, top quark loop contribute to ρ parameter

$$\Delta\rho^{top} = \frac{3\sqrt{2}G_F}{8\pi^2} m_t^2$$

In this model, this leading m_t^2 contribution is suppressed by heavy W_2 mass

\Rightarrow

$$\begin{aligned}\Delta r^{top} &= -\frac{c_w^2}{s_w^2} \Delta\rho \\ &= \frac{3\sqrt{2}G_F}{8\pi^2} c_w^2 \left(\frac{c_w^2}{s_w^2} - 1 \right) \frac{M_{W_1}^2}{M_{W_2}^2 - M_{W_1}^2} m_t^2\end{aligned}$$

For $M_{W_2} = 400\text{GeV}$, the leading top quark contribution in this model is smaller than the $\ln(m_t^2)$ contribution in SM!!

Prediction for top quark mass from oblique corrections is lost!!

Contribution from lightest Higgs: suppressed by heavy gauge boson masses:

$$\Delta r = \frac{\sqrt{2}G_F}{48\pi^2} \left(\frac{M_{W_1}^2}{M_{W_2}^2} \frac{c_w^2}{s_w^2} (1 - 2s_w^2) + \frac{M_{W_1}^2}{M_{Z_2}^2} \frac{1}{s_w^2} (4c_w^2 - 1) \right)$$

\Rightarrow model cannot be trivially ruled out!!

Unitarity Bound on Higgs Mass

Require Higgs self-coupling perturbative up to unification scale:

- With an additional gauge singlet:

Tobe and Wells, 2002

Non-SUSY case:

$$\sin^2 \theta_w = 1/4 \Rightarrow \Lambda = 3.8 \text{ TeV}, m_h < 460 \text{ GeV}$$

$$\sin^2 \theta_w = 3/8 \Rightarrow \Lambda \simeq 10^{13} \text{ GeV}, m_h < 200 \text{ GeV}$$

$$\Lambda = M_{pl} \Rightarrow m_h < 180 \text{ GeV}$$

SUSY case:

$$\sin^2 \theta_w = 1/4 \Rightarrow \Lambda = 37 \text{ TeV}, m_h < 350 \text{ GeV}$$

$$\sin^2 \theta_w = 3/8 \Rightarrow \Lambda \simeq 2 \times 10^{16} \text{ GeV}, m_h < 120 \text{ GeV}$$

- MSSM with an additional triplet:

Espinosa and Quiros, 1998

$$\Lambda \simeq 10^{17} \text{ GeV}, m_h < 205 \text{ GeV}$$

- SM with an additional real triplet ($Y=0$):

Forshaw et al, 2003

$$\Lambda \sim 1 \text{ TeV}, m_h < 520 \text{ GeV}$$

What is the $SU(2)$ Triplet Higgs Good For?

- Neutrino Masses

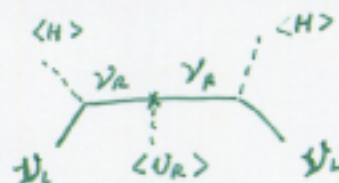
see-saw mechanism:

$$SO(10) \xrightarrow{v_{\text{GUT}}} SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

$$\xrightarrow{v_R} SU(2)_L \times U(1)_Y \Rightarrow \begin{pmatrix} 0 & 0 \\ 0 & f v_R \end{pmatrix}$$

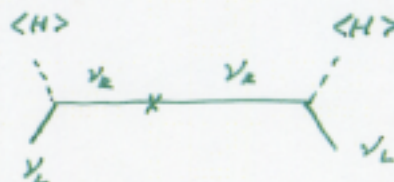
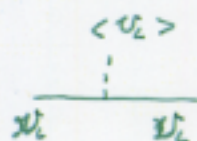
$$\xrightarrow{v_{\text{EW}}} U(1)_{\text{EW}} \Rightarrow \begin{pmatrix} f v_L & h v_{\text{EW}} \\ h v_{\text{EW}} & f v_R \end{pmatrix}$$

— Type I see-saw mechanism: **without parity**



$$\begin{pmatrix} 0 & M_{LR}^T \\ M_{LR} & M_{RR} \end{pmatrix}, \quad M_{\nu}^{\text{eff}} = -M_{LR} M_{RR}^{-1} M_{LR}^T$$

— Type II see-saw mechanism: **with parity**
if there is parity in the model, e.g. Left-Right models, $SO(10)$ models



$$\langle \nu_L \rangle \neq 0$$

$$\nu_L \sim \frac{v_{\text{EW}}^2}{v_R}$$

$$\begin{pmatrix} M_{LL} & M_{LR}^T \\ M_{LR} & M_{RR} \end{pmatrix},$$

$$M_{\nu}^{\text{eff}} = M_{LL} - M_{LR} M_{RR}^{-1} M_{LR}^T$$

$$f \frac{v_{\text{EW}}^2}{v_R} - \frac{v_{\text{EW}}^2}{f v_R}$$

↑ parity symm.

- Bi-Large Mixing Angles

- Family Symmetry: Type I see-saw mechanism
Typically give hierarchical neutrino masses
 $m_{\nu_3} \gg m_{\nu_2} \gg m_{\nu_1}$
- Renormalization Group Enhancement

at GUT scale: starting with leptonic mixing matrix $= V_{CKM}$, nearly degenerate neutrino masses, and identical neutrino Majorana masses

These boundary conditions can be satisfied, if

$$\begin{aligned} M_{LL} &\sim I \cdot v_L &\Rightarrow &\text{degenerate masses} \\ M_{LR} M_{RR}^{-1} M_{LR} &\Rightarrow &\text{mixing matrix} \sim V_{CKM} \end{aligned}$$

- Minimal SO(10) Model with approximate $b - \tau$ unification

both LH and RH Majorana mass terms for neutrinos have identical couplings (thanks to the parity)

for small $\tan \beta$: atmospheric maximal mixing a consequence of $b - \tau$ unification

many natural scenarios require **Type II see-saw mechanism** thus the SU(2) triplet Δ_L having non-zero VEV

(For a review on neutrino masses in SO(10) models, see e.g. M.-C. Chen and K.T. Mahanthappa, hep-ph/0305086)

- Leptogenesis through the decay of the triplet Higgs
Ma and Sarkar, 1998

First generate lepton Asymmetry: Interference between the CP violating decay of $\Delta^{++} \rightarrow l^+ l^+$ at tree level and one-loop:

$$\left(\begin{array}{c} \Delta_1^{++} \\ \swarrow \quad \searrow \\ \ell^+ \quad \ell^+ \end{array} + \begin{array}{c} \Delta_1^{++} \quad \Delta_2^{++} \\ \swarrow \quad \searrow \\ \ell^+ \quad \ell^+ \end{array} \right)$$

(Note: The diagram shows a tree-level decay of Δ_1^{++} into two ℓ^+ and a one-loop diagram where Δ_1^{++} and Δ_2^{++} exchange a h^+ Higgs boson before decaying into two ℓ^+).

L is then converted to B due to EW anomaly

- Strong CP problem, SUSY CP problem:
(Babu, Dutta, Mohapatra, Rasin, Senjanovic)

SUSY Left-Right Model:

$$\bar{\Theta} = \Theta + \text{Arg} \det(M_u M_d) - 3\text{Arg}(M_{\tilde{g}})$$

Θ : coefficient of $F_{\mu\nu} \tilde{F}^{\mu\nu}$ term (P violating)

P is invariant above scale $M_R \Rightarrow \Theta = 0$ above M_R

Left-Right Symmetry \Rightarrow

$$m_{\tilde{g}} = \text{real above } M_R$$

Yukawa coupling constants hermitian

$$\bar{\Theta} = 0 \text{ above } M_R$$

Below $M_R \Rightarrow$ RG corrections must be small so that $\bar{\Theta}$ is kept small

Signatures of the Triplet Higgs

(Gunion, Huitu, Maalampi, Pietila, Raidal, Cuypers,)

Tree level $H^\pm W^\mp Z$ vertex: generally present in models with triplet and/or higher Higgs representations

$$Z \rightarrow H^{++} H^{--}, \quad W^+ W^- \rightarrow H^{--}, \quad \text{current Do bound } m_{\Delta^{--}} > 116 \text{ GeV}$$

- neutral sector
- singly charged sector
- doubly charged sector possible decay channels:
 - $\Delta^{++} \rightarrow e^+ e^+$
 - $\Delta^{++} \rightarrow W^+ W^+$ suppressed as $\langle \Delta_L \rangle \ll 1$
 - $\Delta^{++} \rightarrow h^+ W^+$ suppressed by phase space
- Lepton number violation ($\Delta L = 2$) processes

$$f_{ij} L_{i,L}^T C \tau_2 \Delta_L L_{j,L} =$$

$$f_{ij} \left(\underbrace{\Delta_L^0 \nu_{i,L} \nu_{j,L}}_{\nu\text{-mass}} + \frac{1}{2} \Delta_L^+ [\nu_{i,L} e_{j,L} + e_{i,L} \nu_{j,L}] + \Delta_{LL}^{++} e_{i,L} e_{j,L} \right)$$

leads to $\Delta L = 2$ decay couplings

$$e^- e^- \rightarrow \Delta^{--}, \quad \mu^- \mu^- \rightarrow \Delta^{--}$$

Currently we do not have any limit on $f_{\tau\tau}$

strongest constraints are for f_{ee} and $f_{\mu\mu}$: ($m_{\Delta^{--}}$ in GeV)

- * from Bhabha scattering

$$|f_{ee}|^2 < 10^{-5} m_{\Delta^{--}}^2$$

- * to avoid giving wrong sign contribution to $(g - 2)_\mu$ deviation

$$|f_{\mu\mu}|^2 < 5 \times 10^{-7} m_{\Delta^{--}}^2$$

- * from muonium-anti-muonium conversion:

$$|f_{ee} f_{\mu\mu}| < 10^{-7} m_{\Delta^{--}}^2$$

some weaker constraints:

- * from $\mu^- \rightarrow e^- e^- e^+$

$$|f_{e\mu} f_{ee}| < 10^{-11} m_{\Delta^{--}}^2$$

If $\langle \Delta_L \rangle = 0 \Rightarrow \Gamma_{\Delta^{--}}^T$ small
possibly very large s-channel $e^- e^-$ and $\mu^- \mu^-$ production rates (Gunion, 1998)

can probe very small $f_{ee}, f_{\mu\mu} \sim 10^{-16}$ at $e^- e^-$ collider with $L = 300 \text{ fb}^{-1}$

\Rightarrow relevant range for see-saw

\Rightarrow neutrino physics at the colliders